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# NONSTEADY THERMODIFFUSIONAL SEPARATION OF BINARY MIXTURES UNDER SAMPLING CONDITIONS

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An accurate solution is obtained for the nonlinear nonsteady problem of the thermodiffusional separation of two-component mixtures in sampling conditions.

The process of thermodiffusional separation of binary mixtures, taking account of sampling, is described by the system of equations [1]

$$\frac{\partial c}{\partial \tau}=\frac{\partial}{\partial \xi}\left[\frac{\partial c}{\partial \xi}-2bc(1-c)-2\varkappa c\right],\quad c(\xi,\ 0)=c_0 \tag{1}$$

with the appropriate boundary conditions at the ends of the column. In particular, if the column is closed at one end ($\xi$ = 1), while the other is connected to an infinite reservoir ($\xi$ = 0), these conditions take the form

$$c(0,\ \tau)=c_0,\quad \left[\frac{\partial c}{\partial \xi}-2bc(1-c)\right]_{\xi=1}=0. \tag{2}$$

Here and above, the following dimensionless parameters are used

$$\tau=\frac{Kt}{L^2\mu},\quad \xi=\frac{z}{L},\quad b=\frac{HL}{2K},\quad \varkappa=\frac{\sigma L}{2K}. \tag{3}$$

An important characteristic of separating equipment is the concentration difference at the ends of the columns

$$\Delta c=c(1,\ \tau)-c_0, \tag{4}$$

i.e., it is necessary to know the explicit dependence c(1, $\tau$). The steady value c(1, $\infty$) of this quantity is known [2]; it is found from the following transcendental equation

$$\underset{\tau\to\infty}{c(1,\ \tau)}=c(1,\ \infty)=c_0\,\frac{1+\dfrac{b+\varkappa}{\sqrt{p_0}}\,\mathrm{th}\sqrt{p_0}}{1+\dfrac{2bc_0+\varkappa-b}{\sqrt{p_0}}\,\mathrm{th}\sqrt{p_0}}, \tag{5}$$

$$p_0=(\varkappa+b)^2-4b\varkappa c(1,\ \infty)>0. \tag{6}$$

Although the nonsteady thermodiffusional separation of two-component mixtures forms the subject of an extensive literature (see [1], for example), the solution has been obtained either without taking account of sampling ($\varkappa$=0) or in a linear approximation c(1 − c) ≃ const c. At the same time, methods of linearizing the generalized nonlinear boundary prob-

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lems of the nonsteady separation of binary mixtures were proposed earlier in [3].

Following [3], the solution of the system in Eqs. (1) and (2) may be written in the form

$$c(\xi,\ \tau)=\frac{1}{2b}\left[b+\varkappa+\frac{\psi'_{\xi}}{\psi}\right],$$

$$\psi(\xi,\ \tau)=\frac{1}{2\pi i}\int\limits_{\alpha-i\infty}^{\alpha+i\infty}F(\xi,\ p)\exp(p\tau)\,dp, \tag{7}$$

$$F(\xi,\ p)=\frac{\exp(B\xi)}{p-B^2}-\exp B\,\frac{D\left(\operatorname{ch}\sqrt{p}\,\xi+\frac{B}{\sqrt{p}}\operatorname{sh}\sqrt{p}\,\xi\right)}{(p-B^2)\left[R(\varkappa)\operatorname{ch}\sqrt{p}+\frac{B}{\sqrt{p}}\,R\left(\frac{\varkappa p}{B^2}\right)\operatorname{sh}\sqrt{p}\right]},$$

$$D=B^2+2B\varkappa+\varkappa^2-b^2=4b^2c_0(1-c_0),$$

$$R(Z)=p+2BZ+\varkappa^2-b^2,\quad B=2bc_0-b-\varkappa.$$

Writing the solution of the initial problem in this form allows the influence of sampling on the dynamic characteristics to be investigated with an arbitrary concentration of the enriched component, i.e., taking account of nonlinearity in the system in Eqs. (1) and (2). Let us consider the behavior of $c(\xi,\ \tau)|_{\xi=1}=c(1,\ \tau)$ in more detail. The analysis of singular points of the functions F(1, p) and $F_{\xi}'(1,\ p)$ shows, above all, that $p = p_0$ is a positive pole; $((b+\varkappa)^2>4\varkappa b)$. It is difficult to establish this directly by substituting Eq. (6) into the denominator of Eq. (7), since the result obtained is Eq. (5) for determining the steady value of the concentration. These expressions also have an indeterminate number of negative poles given by the equations

$$\frac{\operatorname{th}\sqrt{p_n}}{\sqrt{p_n}}=-\frac{R(\varkappa)}{BR\left(\frac{\varkappa p_n}{B^2}\right)},\quad n=1,\ 2,\ \ldots \tag{8}$$

Taking account of these properties of the functions F(1, p) and $F_{\xi}'(1,\ p)$ for the concentration, it is found that

$$c(1,\ \tau)=\frac{1}{2b}\left\{b+\varkappa+\frac{\sum\limits_{n=0}^{\infty}\frac{B^2R\left(\frac{\varkappa p_n}{B^2}\right)-(b^2-\varkappa^2)R(\varkappa)}{\Delta}\exp[(p_n-p_0)\tau]}{\sum\limits_{n=0}^{\infty}\frac{BR\left(\frac{\varkappa p_n}{B^2}\right)-(B+2\varkappa)R(\varkappa)}{\Delta}\exp[(p_n-p_0)\tau]}\right\}, \tag{9}$$

where

$$\Delta=BR\left(\frac{\varkappa p_n}{B^2}\right)\left[1+\frac{BR\left(\frac{\varkappa p_n}{B^2}\right)}{2p_n}\right]-R(\varkappa)\left[B+2\varkappa-\frac{BR\left(\frac{\varkappa p_n}{B^2}\right)}{2p_n}+\frac{R(\varkappa)}{2}\right]. \tag{10}$$

Hence a series of well-known results may be easily obtained. In particular, as $\tau \to \infty$, Eq. (9) transforms to the steady solution in Eq. (5). As $\varkappa \to 0$, Eq. (9) yields the result corresponding to nonsampling conditions [1, 4]. Finally, when $c_0 \ll 1$ and $c(1,\ \infty)\ll 1$, the solution in the linear approximation is found [1].

As is known, expansions of the type in Eq. (9) with respect to the poles of the integrand only give satisfactory results at sufficiently large times $(p_0-p_1)\tau\gg 1$. At small times, however, such series are poorly convergent and the computation procedure is significantly complicated. At the same time, the region of small times corresponds to significantly unstable conditions. Therefore, more detailed study of the behavior of the concentration difference at the ends as $\tau \to 0$ is of much interest. In this case, the main contribution comes from the region $|p| \gg 1$, where

$$F(\xi, p) \underset{|p|\gg 1}{\simeq} \frac{\exp(B\xi)}{p - B^2} - \frac{D \exp[B + \sqrt{p}(\xi - 1)]}{(p - B^2)(\sqrt{p} + \varkappa + b)(\sqrt{p} + \varkappa - b)} \tag{11}$$

and the concentration at the end of the column is

$$\begin{gathered} c(1, \tau) = c_0(bc_0 - b - \varkappa)\{B\exp(B^2\tau)\operatorname{erfc}(B\sqrt{\tau}) + (b - \varkappa) \times \\ \times \exp[(\varkappa - b)^2\tau]\operatorname{erfc}[(\varkappa - b)\sqrt{\tau}]\} : \{-B\varkappa\exp(B^2\tau)\operatorname{erfc}(B\sqrt{\tau}) - \\ -(1 - c_0)(b + \varkappa)(bc_0 - \varkappa)\exp[(b + \varkappa)^2\tau]\operatorname{erfc}[(b + \varkappa)\sqrt{\tau}] + \\ + c_0(b - \varkappa)(bc_0 - b - \varkappa)\exp[(\varkappa - b)^2\tau]\operatorname{erfc}[(\varkappa - b)\sqrt{\tau}]\}. \end{gathered} \tag{12}$$

Substitution of Eq. (12) into Eq. (4) gives, in the limit as $\tau \to 0$

$$\Delta c \underset{\tau \to 0}{\simeq} 4bc_0(1 - c_0)\sqrt{\frac{\tau}{\pi}}. \tag{13}$$

From the analysis of Eqs. (8)-(12), it is evident that the attainment of a specific concentration at the end of the column requires more time in the presence of sampling than in the case of column operation with no sampling; however, in the initial stage, as is evident from Eq. (13), its influence is not felt.

Finiteness of the initial concentration leads to increase in the characteristic times of the process in comparison with the linear ($c_0 \ll 1$) case.

## NOTATION

H, K, $\sigma$, transfer coefficients; $\mu$, mass of the material per unit length; L, column length; c, concentration of the enriched component; $c_0$, initial concentration.

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